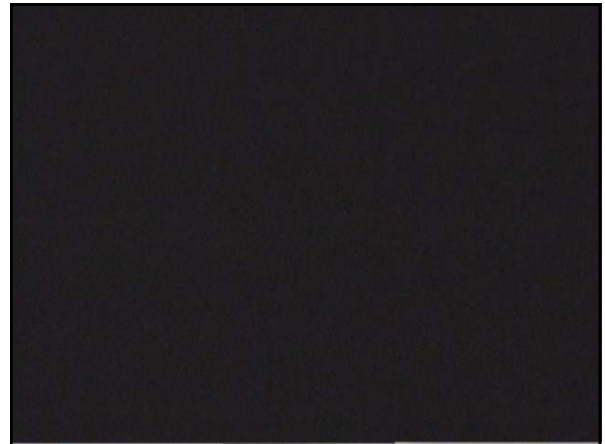


Movie Segment

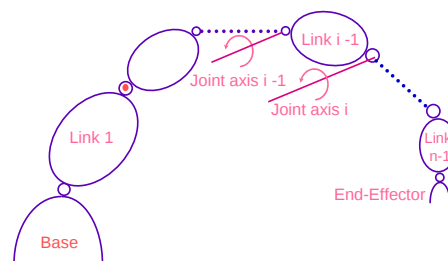
The Hummingbird, IBM Watson Research Center, ICRA 1992 video proceedings



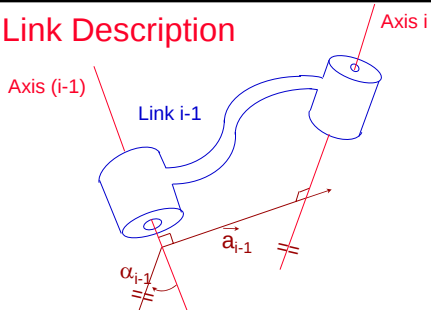
Manipulator Kinematics

- Link Description
- Denavit-Hartenberg Notation
- Frame Attachment
- Forward Kinematics

Manipulator



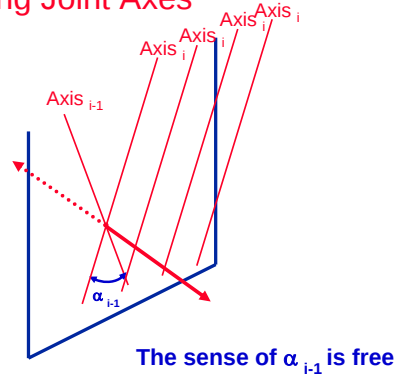
Link Description



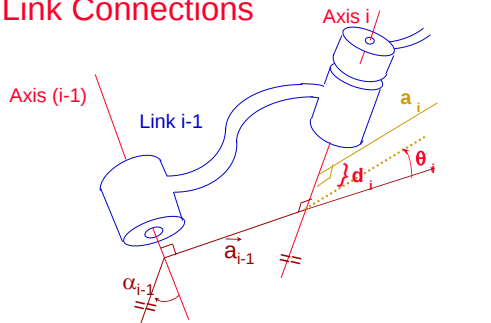
a_{i-1} : Link Length - mutual perpendicular
unique except for parallel axis

α_{i-1} : Link Twist - measured in the right-hand sense about $\vec{a}_{i-1}$

Intersecting Joint Axes

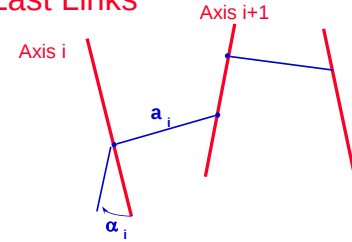


Link Connections



d_i : Link Offset -- variable if joint i is *prismatic*
 θ_i : Joint Angle -- variable if joint i is *revolute*

First & Last Links



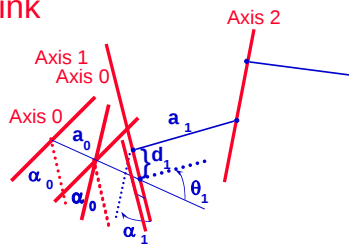
a_i and α_i depend on joint axes i and $i+1$

Axes 1 to n : **determined**

➡ $a_1, a_2, \dots, a_{n-1}$ and $\alpha_1, \alpha_2, \dots, \alpha_{n-1}$

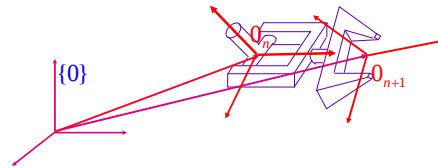
Convention: $a_0 = a_n = 0$ and $\alpha_0 = \alpha_n = 0$

First Link

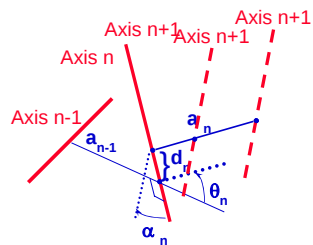


$a_0 = 0$ and $\alpha_0 = 0$

End-Effector Frame

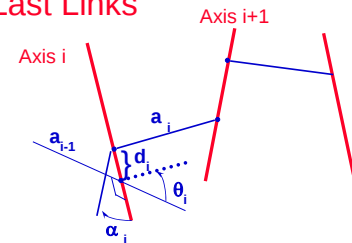


Last Link



$a_n = 0$ and $\alpha_n = 0$

First & Last Links

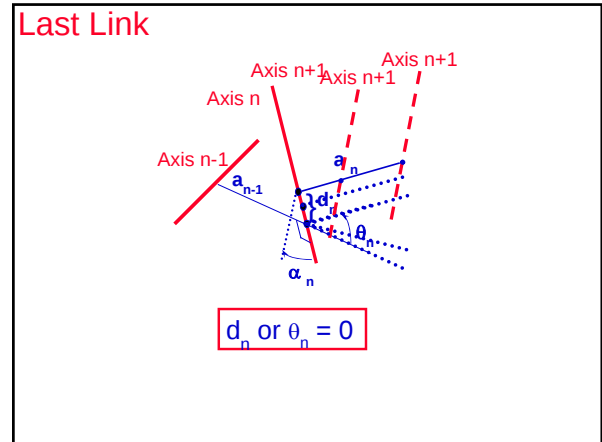
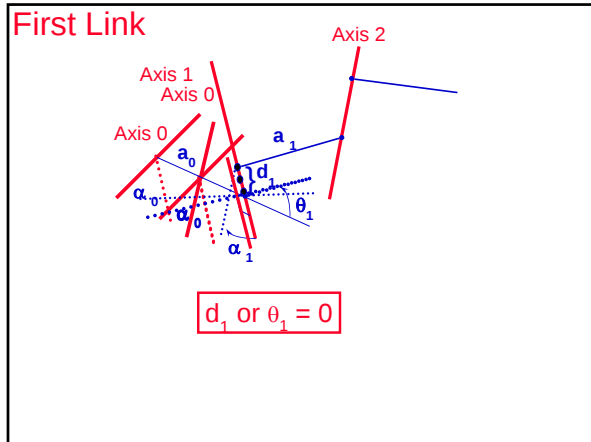


θ_i and d_i depend on links $i-1$ and i

➡ $\theta_2, \theta_3, \dots, \theta_{n-1}$ and $d_2, d_3, \dots, d_{n-1}$

Convention: set the constant parameters to zero

Following joint type: d_1 or $\theta_1 = 0$ and d_n or $\theta_n = 0$



Denavit-Hartenberg Parameters

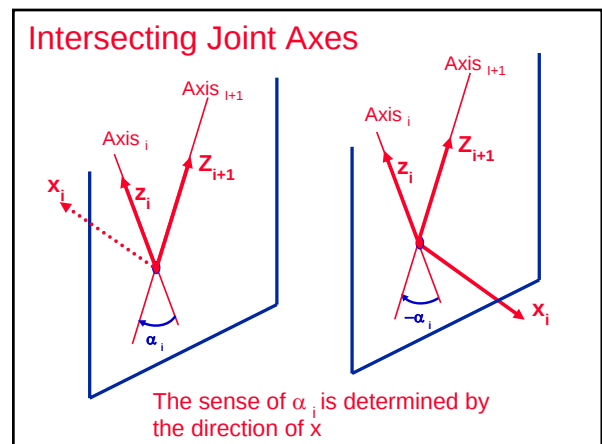
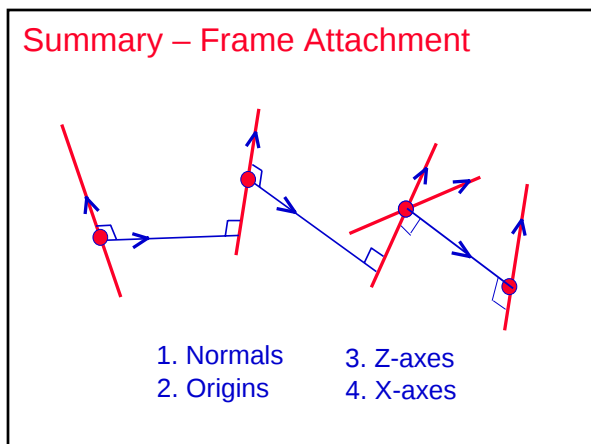
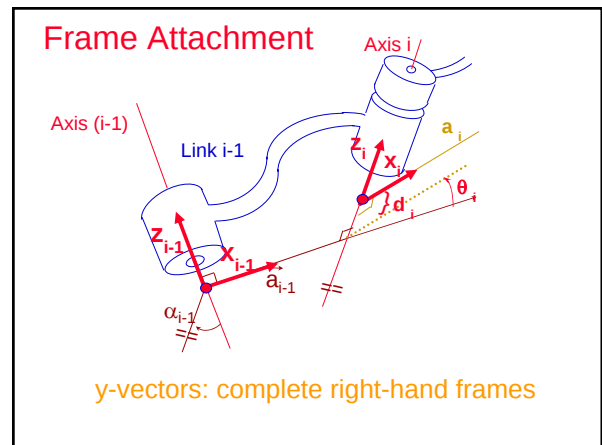
4 D-H parameters (α_i , a_i , d_i , θ_i)

3 fixed link parameters

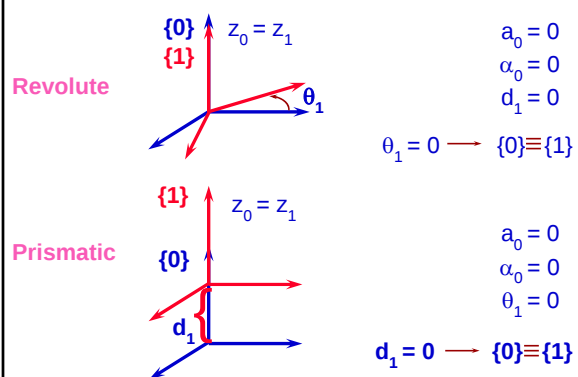
1 joint variable $\begin{cases} \theta_i \text{ revolute joint} \\ d_i \text{ prismatic joint} \end{cases}$

α_i and a_i : describe the Link i

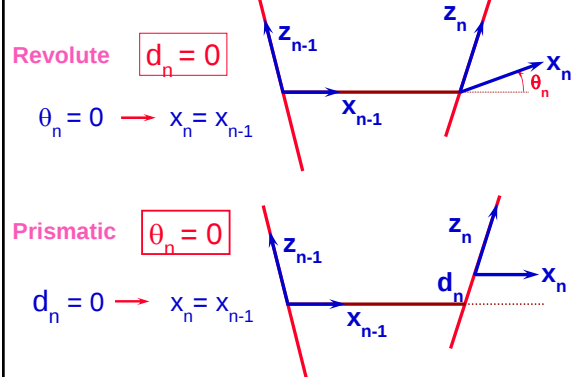
d_i and θ_i : describe the Link's connection



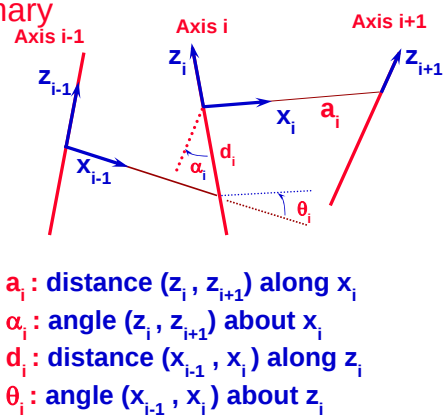
First Link



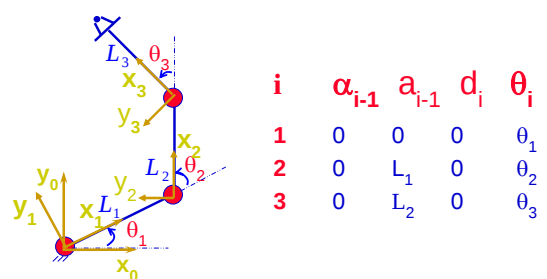
Last Link



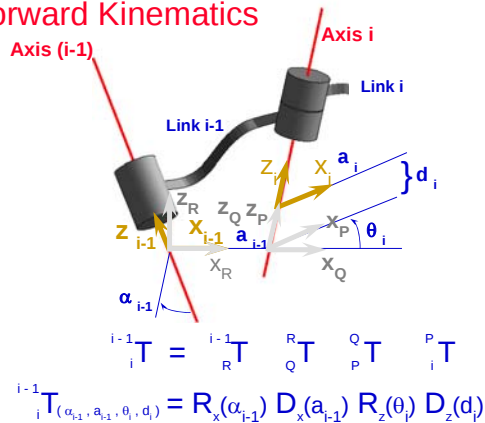
Summary



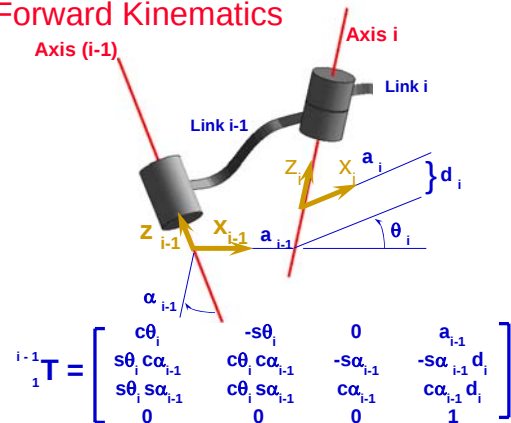
Example – RRR Arm

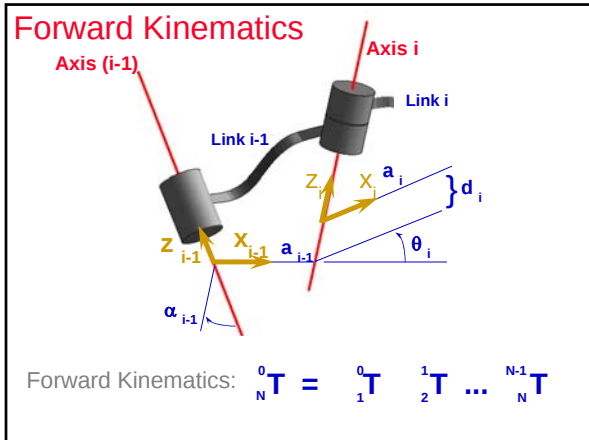


Forward Kinematics



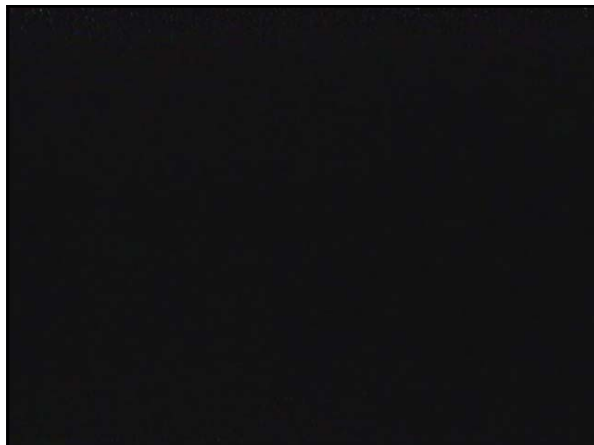
Forward Kinematics



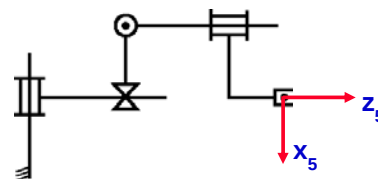


Movie Segment

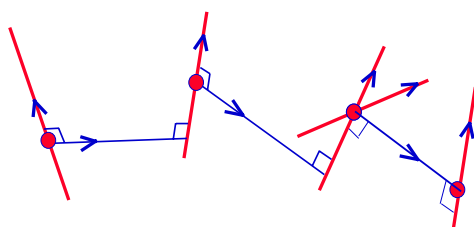
Brachiation Robot, Nagoya University, ICRA 1993 video proceedings



Example - RPRR

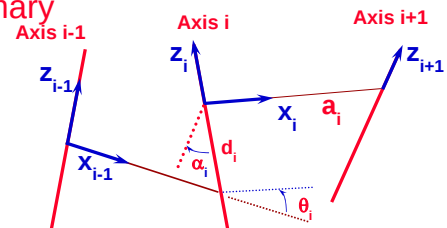


Summary – Frame Attachment



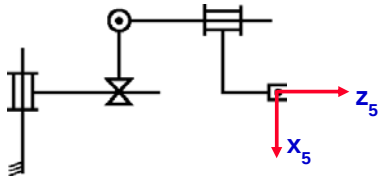
1. Normals
2. Origins
3. Z-axes
4. X-axes

Summary

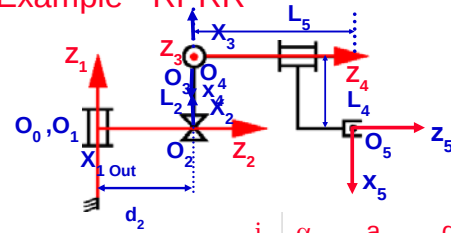


- a_i : distance (z_i, z_{i+1}) along x_i
- α_i : angle (z_i, z_{i+1}) about x_i
- d_i : distance (x_{i-1}, x_i) along z_i
- θ_i : angle (x_{i-1}, x_i) about z_i

Example - RPRR



Example - RPRR



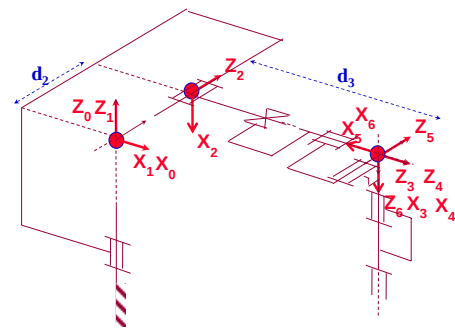
i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	-90
3	-90	L_2	0	θ_3
4	90	0	L_5	θ_4
5	0	L_4	0	0

a_i : distance (z_i, z_{i+1}) along x_i
 α_i : angle (z_i, z_{i+1}) about x_i
 d_i : distance (x_{i-1}, x_i) along z_i
 θ_i : angle (x_{i-1}, x_i) about z_i

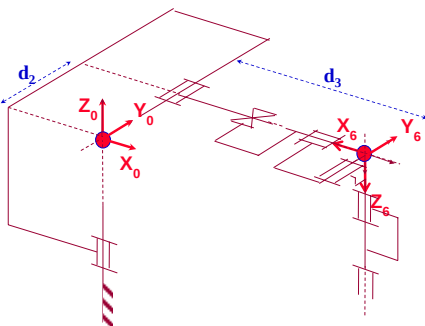
Stanford Scheinman Arm



Stanford Scheinman Arm

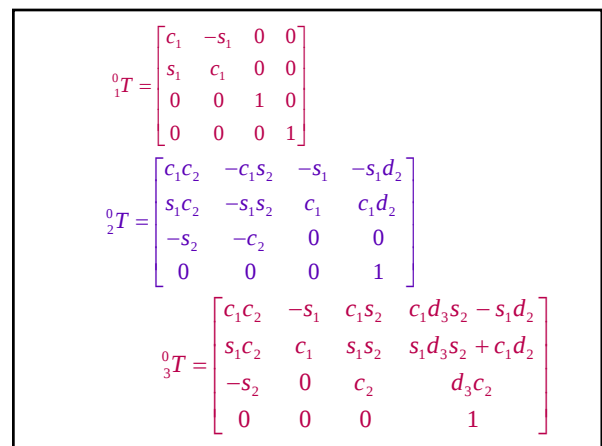
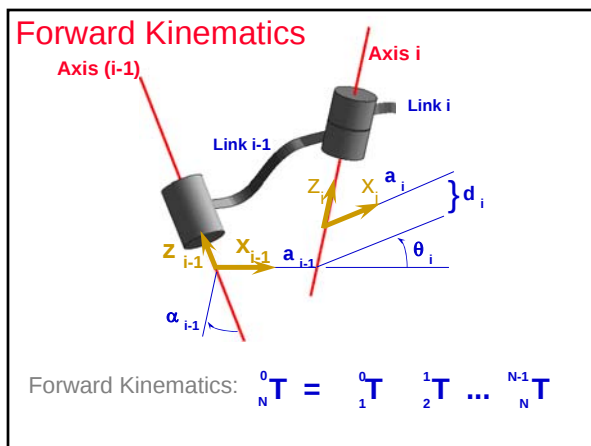
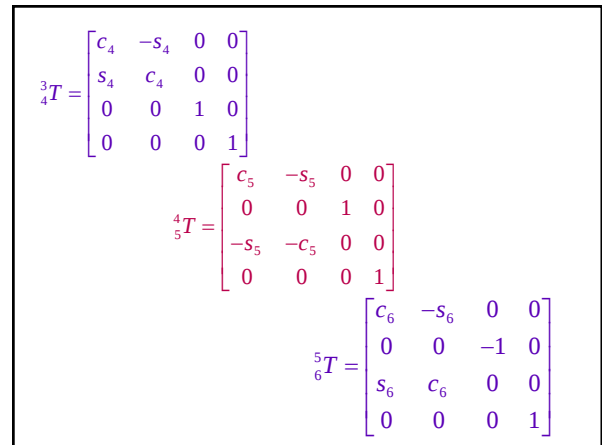
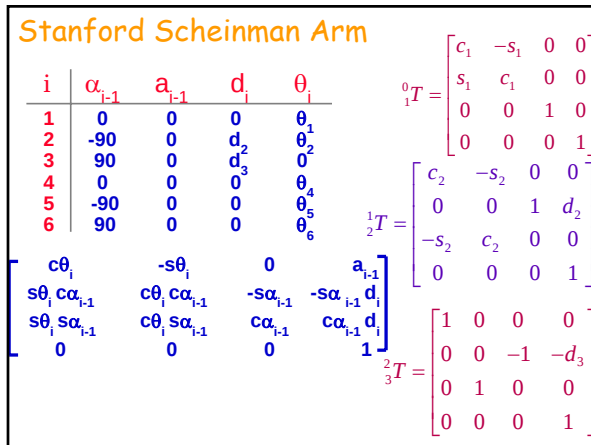
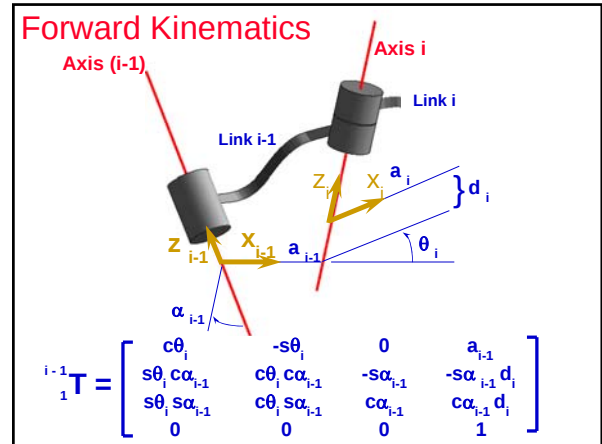
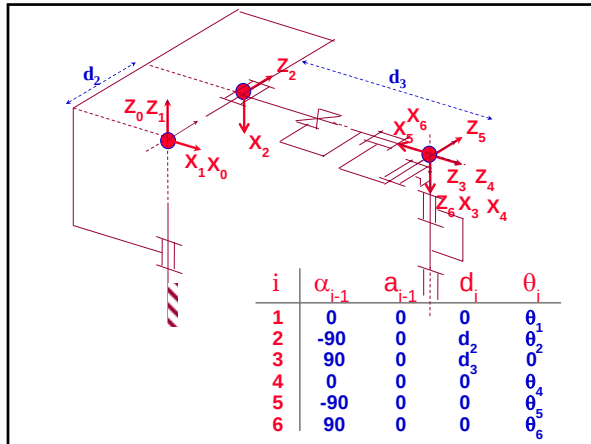


Stanford Scheinman Arm



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	0	d_2	θ_2
3	90	0	0	0
4	0	0	d_3	θ_4
5	-90	0	0	θ_5
6	90	0	0	θ_6

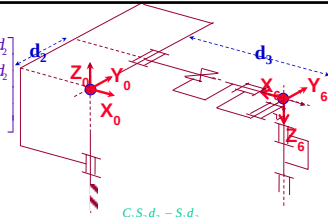
a_i : distance (z_i, z_{i+1}) along x_i
 α_i : angle (z_i, z_{i+1}) about x_i
 d_i : distance (x_{i-1}, x_i) along z_i
 θ_i : angle (x_{i-1}, x_i) about z_i



$${}^0_4T = \begin{bmatrix} c_1c_2c_4 - s_1s_4 & -c_1c_2s_4 - s_1c_4 & c_1s_2 & c_1d_3s_2 - s_1d_2 \\ s_1c_2c_4 + c_1s_4 & -s_1c_2s_4 + c_1c_4 & s_1s_2 & s_1d_3s_2 + c_1d_2 \\ -s_2c_4 & s_2s_4 & c_2 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_5T = \begin{bmatrix} X & X & -c_1c_2s_4 - s_1c_4 & c_1d_3s_2 - s_1d_2 \\ X & X & -s_1c_2s_4 + c_1c_4 & s_1d_3s_2 + c_1d_2 \\ X & X & s_2s_4 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$${}^0_6T = \begin{bmatrix} X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\ X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\ X & X & -s_2c_4s_5 + c_2c_5 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



$${}^0_6T = \begin{bmatrix} X & X & c_1c_2c_4s_5 - s_1s_4s_5 + c_1s_2s_5 & c_1d_3s_2 - s_1d_2 \\ X & X & s_1c_2c_4s_5 + c_1s_4s_5 + s_1s_2c_5 & s_1d_3s_2 + c_1d_2 \\ X & X & -s_2c_4s_5 + c_2c_5 & d_3c_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$x = \begin{bmatrix} x_p \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} C_1S_2d_3 - S_1d_2 \\ S_1S_2d_3 + C_1d_2 \\ C_2d_3 \\ C_1[C_2(C_4C_5C_6 - S_2S_6) - S_2S_2C_6] - S_1(S_2C_5C_6 + C_4S_6) \\ S_1[C_2(C_4C_5C_6 - S_2S_6) - S_2S_2C_6] + C_1(S_2C_5C_6 + C_4S_6) \\ -S_2(C_4C_5C_6 - S_2S_6) - C_2S_5C_6 \\ C_1[-C_2(C_4C_5S_6 + S_2C_6) + S_2S_2S_6] - S_1(-S_2C_5S_6 + C_4C_6) \\ S_1[-C_2(C_4C_5S_6 + S_2C_6) + S_2S_2S_6] + C_1(-S_2C_5S_6 + C_4C_6) \\ S_2(C_4C_5S_6 + S_2C_6) + C_2S_5S_6 \\ C_1(C_2C_4S_5 + S_2C_5) - S_1S_2S_5 \\ S_1(C_2C_4S_5 + S_2C_5) + C_1S_2S_5 \\ -S_2C_4S_5 + C_2C_5 \end{bmatrix}$$